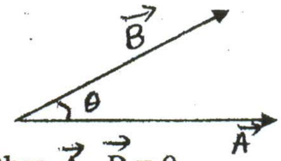
Sharjah Indian School, Sharjah

Notes on "WORK, ENERGY & POWER"

Scalar Product (Dot product) :- If the product of two vectors is a scalar, it is said to be a scalar product. The scalar or dot product of two vectors is defined as the product of the magnitude of two vectors and the cosine of the angle between them. If $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, then the scalar or dot product is given by,

$$\vec{A} . \vec{B} = AB \cos\theta$$



Properties of dot product

(i) It is commutative. i.e., $\vec{A} . \vec{B} = \vec{B} . \vec{A}$

(ii) When vectors are perpendicular to each other, $\vec{A} . \vec{B} = 0$.

(iii) When vectors are parallel to each other, $\vec{A} . \vec{B} = AB$

(iv) $\vec{A} . \vec{A} = A^2$

(v) $\hat{i} . \hat{i} = \hat{j} . \hat{j} = \hat{k} . \hat{k} = 1$ & $\hat{i} . \hat{j} = \hat{j} . \hat{k} = \hat{k} . \hat{i} = 0$

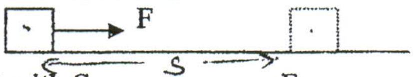
WORK

Work is said to be done, if the point of application of the force is displaced. Work done is measured as the product of the displacement and the force applied in the direction of the displacement.

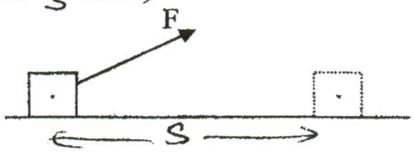
Work done by a constant force

When F and S are in the same direction,

$W = F S$



When F makes an angle θ with S,



$W = F_x S$

i.e., $W = FS \cos\theta$

$W = \vec{F} . \vec{S}$

The S.I unit of work is Joule (J). Work is said to be 1 Joule, when s force of 1N acts on body and displaces it through 1m in its own direction.

Dimension of work is ML^2T^{-2}

[**Special notes**: (i) Work done is positive when $\theta < 90^\circ$. e.g. work done by gravity on freely falling body. (ii) Work done is negative when θ lies between 90° and 180°. e.g. Work done by friction. (iii) Work done is zero when $\theta = 90^\circ$ or $S = 0$ or $F = 0$. e.g Work done by centripetal force, work done by a person carrying a load on his head and walks in a platform, etc.]

Work done by a variable force

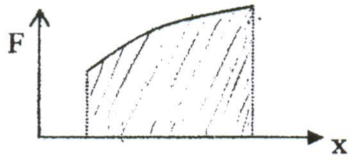
When a body is displaced under force F that keeps changing, the work done by the force can be determined in the following ways:

(i) Mathematically,

By the method of integration, $W = \int F\, dx$

(ii) Graphically,

By determining the area enclosed by force-displacement graph.



(for proof, refer: vel – time graph)

Energy of a body is the ability to do work. It is a scalar quantity. S.I unit is Joules.

Mechanical energy is of two different types: (i) Kinetic energy and (ii) potential energy.

Kinetic energy (K.E) : Energy possessed by a body by virtue of its motion. E.g. energy of blowing wind, flowing water, fired bullet etc.

Expression for K.E

(i) Under the action of constant force:

Consider a constant force F acting on a body at rest. Let the body move through a distance S in the direction of force changing its velocity to 'v', with acceleration 'a'. Then,

$V^2 = u^2 + 2aS$ i.e, $v^2 = 2aS$ $S = v^2 / 2a$

Force $F = ma$

Work done $(W) = F S = ma\, v^2 / 2a$

So, $K.E = ½ mv^2$.

(ii) Under the action of a variable force: (By the method of calculus)

$W = \int F\,dx = \int ma\,dx = \int m\,dv/dt\,dx = \int m\,v dv$ $(v = dx/dt)$

So, $K.E = \int_0^v m\,v dv = m\,[v^2/2]_0^v = ½\,mv^2$

Work-Energy Theorem:- The work done by a force acting on a body is equal to the change in the K.E of the body.

Proof:- Let the variable force F displaces a body of mass 'm' such that the velocity changes from 'u' to 'v'. Then the work done,

$W = \int F\,dx = \int ma\,dx = \int m\,dv/dt\,dx = \int_u^v m\,v dv$ $(v = dx/dt)$

So, Work $= \int_u^v m\,v dv = m\,[v^2/2 - u^2/2] = ½\,mv^2 - ½\,mu^2$.

i.e., Work done = the change in K.E

Potential Energy:- The energy possessed by a body by virtue of its position or shape. It is of different types such as, gravitational P.E, the energy possessed by a stretched spring e.t.c.]

Gravitational P.E:- It is the energy possessed by a body under the gravitational influence of the earth.

Expression:- Gravitational P.E = the work done on a body to lift it through a vertical height 'h'

$= F\,S$

Gravitational P.E $= mgh.$ $(F = mg)$

P.E stored in a spring (Elastic P.E)

When a spring is stretched through a distance 'x', the work is done under variable force. Here, $F \propto x$ or $\mathbf{F = kx}$, where 'k' is a constant called spring constant.

[Unit of k – N/m - the higher the value of k, the stiffer would be the spring.]

So, the work done in stretched spring $W = \int F\,dx = \int_0^x kx\,dx = k\,[x^2/2]_0^x$

Hence P.E stored in the spring $= ½\,kx^2$.

Graphical Method:

The graph between force and displacement of a stretched spring is shown below:

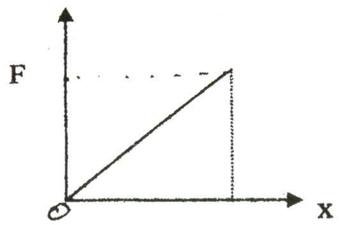
The work done in stretching the spring = the area enclosed by the graph.

i.e., W = the area of the triangle OBC

$= ½\ x\,F = ½\ x\ kx$

So, $W = ½\,kx^2$.

Thus, **P.E stored in the spring = ½ kx²**

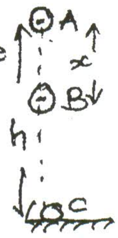


Law of Conservation of Energy: It states that energy can neither be created, not be destroyed. But it can be changed from one form into another.

Proof:

1. In the case of a freely falling body.

(Qn. Show that, for a freely falling body, the total energy is conserved throughout its motion)



Consider a body of mass 'm' situated at a height 'h' above the ground. Since the body is at rest the energy is completely potential.

At the point A

P.E = mgh K.E = 0

Total energy = P.E + K.E = mgh ....(1)

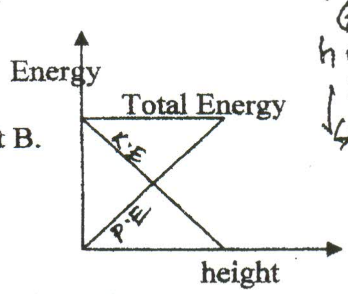
At the point B - at a distance 'x' below A. Let v_1 be the velocity at B.

$P.E = mg(h - x)$ $K.E = ½\,m\,v_1^2$

Total energy = P.E + K.E $= mg(h - x) + ½\,m\,v_1^2$

$[v^2 = u^2 + 2as\ ;\ v_1^2 = 2gx\,]$ $= mgh - mgx + ½\,m\,2gx$

$= mgh$ ....(2)



At the point C (When it just touches the ground) Let v_2 be the velocity at C.

P.E = 0 $K.E = ½\,mv_2^2 = ½\,m\,.\,2gh = mgh$

Total energy = P.E + K.E = 0 + mgh

= mgh ....(3)

From equations (1), (2) and (3), it can be seen that the total energy is always conserved.