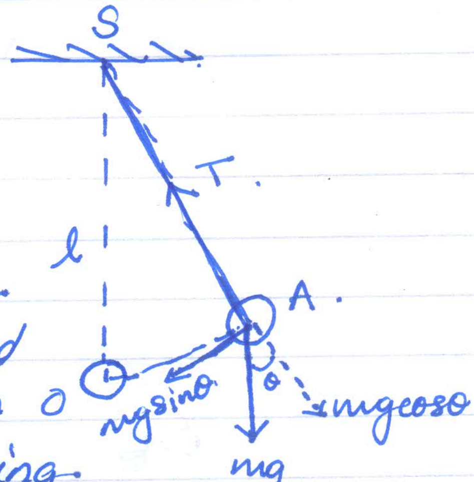
Examples of SHM.

1. Simple pendulum:- A simple pendulum consists of a metallic bob suspended from a long thread whose one end is fixed to a rigid support. SO is the equilibrium position. When the pendulum is displaced through the angle θ with the vertical, it executes oscillations with time period T.

Q. Show that the oscillations of a simple pendulum are in simple harmonic motion. Obtain expression for time period T of the pendulum.

Consider a simple pendulum of length l with a bob of mass m. Let it be displaced through a small angle θ with the vertical as shown.



Let $OA = x$.

At the displaced position, OA, the forces acting on mass m, is, 1. the weight mg.

2. Tension T in the string.

The weight mg can be resolved as shown. The component $mg\cos\theta$ balances the tension T in the string. $mg\sin\theta$ provides necessary restoring force.

$$F = -mg\sin\theta.$$

$$ma = -mg\theta. \qquad \text{For small angles } \sin\theta \simeq \theta.$$

$$a = -g\frac{x}{l}. \qquad \text{Also } \theta = x/l.$$

$$\frac{d^2x}{dt^2} + \frac{g}{l}x = 0 \quad (1)$$ which is similar to the differential equation of SHM,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad (2).$$

Hence the oscillations of a simple pendulum are simple harmonic in nature.

Comparing ① and ②,

$$\omega^2 = g/l. \qquad \omega = \sqrt{g/l}$$

$$\frac{2\pi}{T} = \sqrt{g/l}.$$

$$T = 2\pi\sqrt{l/g} \qquad \nu = \frac{1}{2\pi}\sqrt{g/l}$$

ie, Time period is independent of mass of the bob.

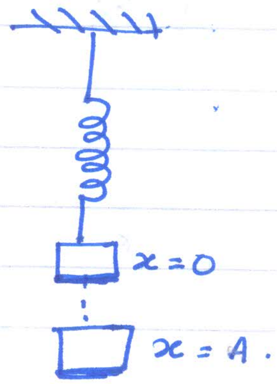
If T_1 and T_2 be the time period of a simple pendulum in two different planets with acceleration due to gravity g_1 & g_2,

$$T_1^2 = \frac{4\pi^2 l}{g_1}, \quad T_2^2 = \frac{4\pi^2 l}{g_2}.$$

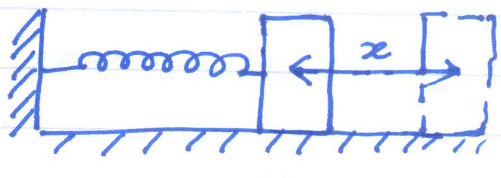
$$\frac{T_1^2}{T_2^2} = \frac{4\pi^2 l}{g_1} \times \frac{g_2}{4\pi^2 l} = \frac{g_2}{g_1}$$

$$\frac{T_1}{T_2} = \sqrt{g_2/g_1}$$

2. Oscillations due to a spring.



Consider a spring of spring constant k. fixed to a support as shown. The other end is connected to a mass m. The body is placed on a smooth frictionless surface.



When the body is displaced and released, it oscillates with frequency ν.

The restoring force,

$$F \propto -x$$

$$F = -kx.$$

$$m \cdot \frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \text{—(1)}$$ which is similar to the expression, $\frac{d^2x}{dt^2} + \omega^2 x = 0$ —(2).

Hence the oscillations are in SHM.

$$\omega^2 = \frac{k}{m} \qquad \omega = \sqrt{k/m}.$$

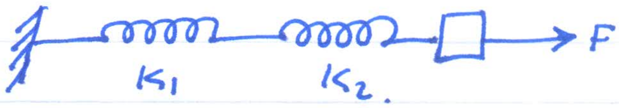
$$2\pi\nu = \sqrt{k/m}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Time period $T = 2\pi\sqrt{\frac{m}{k}}$

Combination of springs.

a. Series combination of springs with spring constants k_1 and k_2.



Here F is the same for both springs. Let x_1 and x_2 be the extensions, $F = -k_1x_1$, $F = -k_2x_2$.

$$x_1 = \frac{-F}{k_1}, \quad x_2 = -F/k_2.$$

Total extension $x_1 + x_2 = -F\left[\frac{1}{k_1} + \frac{1}{k_2}\right]$

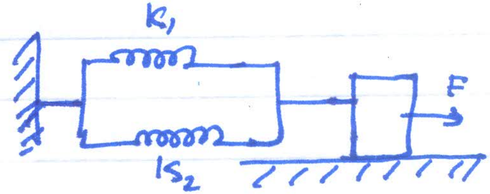
$$x = -F\frac{(k_1+k_2)}{k_1k_2}$$

$$F = -\left(\frac{k_1k_2}{k_1+k_2}\right)x.$$

Net spring constant of the combination,

$$k_s = \frac{k_1k_2}{k_1+k_2}$$

Parallel combination of springs.



Consider two springs of spring constants K_1 and K_2 connected as shown.

Here extension in both the springs are same

$$F_1 = -K_1 x \quad \& \quad F_2 = -K_2 x.$$

$$F_1 + F_2 = -(K_1 + K_2)\,x.$$

$$F = -K_p x. \qquad \underline{\underline{K_p = K_1 + K_2}}.$$

$\therefore$ Timeperiod $T = 2\pi\sqrt{\dfrac{m}{K_p}}.$

$$= 2\pi\sqrt{\frac{m}{K_1 + K_2}}.$$