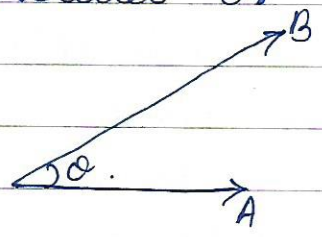
SHARJAH INDIAN SCHOOL (B.W) Grade XI

Rotational Motion and Systems of particles.

Cross product or vector product.

If the product of two vectors is another vector, it is said to be cross product or vector product.

$\vec{A} \times \vec{B} = |A||B| \sin\theta \, \hat{n}$



where $\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$.

It gives the direction of $\vec{A} \times \vec{B}$.

Right Handed screw rule - It gives the direction of $\vec{A} \times \vec{B}$.

Statement - Rotate a right handed screw from $\vec{A}$ to $\vec{B}$ through the smaller angle between them. Then the direction of advancement of the screw gives the direction of $\vec{A} \times \vec{B}$.

If the two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$.

then

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = i(A_y B_z - B_y A_z) - j(A_x B_z - A_z B_x) + k(A_x B_y - A_y B_x)$$

* unit vector perpendicular to both A and B is,

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

Properties of vector product

1) $\vec{A} \times \vec{A} = A.A \sin 0 = 0$
2. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
3. $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$
4. $\vec{A} \times \vec{B} = 0$ if A is parallel to B.
5. $\vec{A} \times \vec{B} = AB$ if A is perpendicular to B.
6. $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$
7. The cross product of two vectors represents the area of the parallelogram formed by them.

Torque (τ)

It is the rotational analogue of force. It is measured as the product of the magnitude of force and the perpendicular distance of the line of action of force from the axis of rotation.

Torque = Force × perpendicular distance.

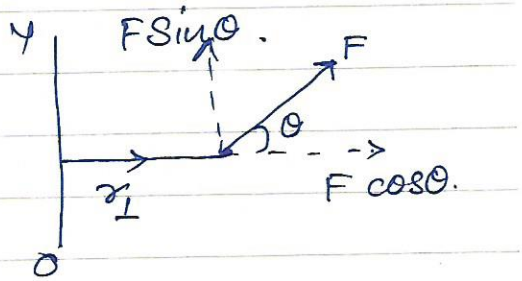
$= F \cdot r_\perp$ or

$= F_\perp r$.

$= F \sin\theta \cdot r$

$= rF \sin\theta$

$\vec{\tau} = \vec{r} \times \vec{F}$



It is a vector quantity. S.I. Unit is Nm.
Dimension is ML^2T^{-2} same as that of work.
Torque due to a force is maximum when.
1. r is maximum - In our experience a door can be opened by applying force at the edge of the door.
2. $\sin\theta$ is maximum ie $\theta = 90°$..