Errors

The error in the measurement of a physical quantity is defined as the difference bet. the true value and the measured value of the physical quantity

Error = True value ~ Measured value

Types of errors – Errors are classified as ① systematic errors ② random errors

Systematic errors – The errors which occurs in a definite pattern is known as systematic error. It can be due to faulty instruments, limitations of an observer, or due to change in conditions of environment.

② Random error – These errors are due to unknown reasons and are not constant in magnitude.

The errors can be reduced by taking a large no. of observations. Then the arithmetic mean of the observations gives the true value of measured physical quantity.

Let a_1, a_2, a_3 --- a_n be the several measurements taken.

True value, $a_{mean} = \frac{a_1 + a_2 + --- + a_n}{n}$.

Absolute error, relative error, % error.

The mag. of the difference between the individual measurement and the true value of the quantity is called absolute error. Denoted as Δa

$\Delta a_1 = a_1 - a_{mean}$

$\Delta a_2 = a_2 - a_{mean}$.

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$\Delta a_n = a_n - a_{mean}$.

The arithmetic mean of all absolute errors is taken as the mean absolute error.

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \cdots + \Delta a_n}{n}$$

$\therefore$ True value $a = a_{mean} \pm \Delta a_{mean}$.

$= P.Q \pm$ Error (M.A.E)

<u>Relative error</u> It is the ratio of mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_{mean}}.$$

When the relative error is expressed in per cent, it is called percentage error.

$$\text{percentage error } \delta a = \left(\frac{\Delta a_{mean}}{a_{mean}}\right) \times 100\%$$

$$= \text{relative error} \times 100\%$$

In an expt the value of R.I of glass was found to be 1.54, 1.53, 1.44, 1.54, 1.56, 1.45 in successive measurements. Cal. ① Mean value of R.I of glass. ② absolute error ③. mean absolute error and ④ Relative error and % error. Also express the result in terms of absolute error and % error.

① True value of R.I $= \frac{a_1 + a_2 + \cdots}{n}$

$$= \frac{1.54 + 1.53 + 1.44 + 1.54 + 1.56 + 1.45}{6}$$

$= 1.51$

②. absolute error = True value − a_1

$\Delta a_1 = 1.51 - 1.54 = -0.03$.

$\Delta a_2 = 1.51 - 1.53 = -0.02$.

$\Delta a_3 = 1.51 - 1.44 = +0.07$

$\Delta a_4 = 1.51 - 1.54 = -0.03$.

$\Delta a_5 = 1.51 - 1.56 = -0.05$,

$= 1.51 - 1.45 = +0.06$.

③ Mean absolute error =

$$\Delta a_{mean} = \frac{0.03 + 0.02 + 0.07 + 0.03 + 0.05 + 0.06}{6}.$$

$$= \frac{0.26}{6} = 0.04$$

③. Relative error $= \frac{\Delta a_{mean}}{a} = \frac{0.04}{1.51}$.

$= 0.02649 = 0.03$

④. % error in $\mu = 0.03 \times 100 = 3\%$

⑤ μ expressed in absolute error

$\mu = 1.51 \pm 0.04$.

⑥ μ expressed in % error $\mu = 1.51 \pm 3\%$

Combination of errors.

a. Error of a sum or diff.

If $Z = A + B$, or $Z = A - B$, and ΔA & ΔB are the errors in A and B,

max error in Z, $\Delta Z = \Delta A + \Delta B$.

ie, when two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in individual quantities.

b. Error of a product or a quotient.

$Z = A \cdot B$, error in A and B are ΔA and ΔB.

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}.$$

when two quantities are multiplied or divided, the relative error in the result is the sum of relative errors in the multipliers.

c. Error in case of measured quantity raised to a power.

$$Z = A^2.$$

$$\frac{\Delta Z}{Z} = 2\frac{\Delta A}{A}.$$

In general, $Z = \frac{A^4 B^{1/3}}{C D^{3/2}}$.

$$\frac{\Delta Z}{Z} = 4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$$

Q. The l and b of a rectang. lamina are measured to be (2.3 ± 0.2) cm and (1.6 ± 0.1) cm. Cal. the area of the lamina with error limits.

$A = l \times b = 2.3 \times 1.6 = 3.68 \text{ cm}^2$.

$$\frac{\Delta A}{A} = \pm\left[\left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta b}{b}\right)\right]$$

$$= \pm\left[\frac{0.2}{2.3} + \frac{0.1}{1.6}\right] = \pm[0.08695 + 0.0625].$$

10mm = 1cm Q. Cal the % error in the determination of $g = 4\pi^2 (l/T^2)$ where l & T are measured with $\pm 2\%$ and $\pm 3\%$ errors resply.

$$\frac{\Delta g}{g}\% = \frac{\Delta l}{l}\% + 2\frac{\Delta T}{T}\%$$

$$= 2\% + 2\times 3\% = 2 + 6 = 8\%$$

X. Det. the max % error in the determination of g due to gravity using the data given.

$l = 100.1$ cm, $R = 2.52$ cm $T = 2.1$ s. The length of the thread is measured with a metre scale of l.c 1mm, r. of the bob is meas. with v-c of l.c 0.01cm and T. is meas. using a stop watch of l.c 0.1s.

$$g = 4\pi^2 \left(\frac{l}{T^2}\right). \qquad l = l' + R.$$

$$= 4\pi^2 \left(\frac{l' + R}{T^2}\right) =$$

$$\frac{\Delta g}{g}\times 100\% = \frac{\Delta l}{l'}\times 100 + \frac{\Delta R}{R}\times 100\% + 2\frac{\Delta T}{T}\times 100\%$$

$$= \frac{1\times 10^{-1}}{100.1}\times 100 + \frac{0.01}{2.52}\times 100 + 2\times\frac{0.1}{2.1}\times 100$$

$$= 0.0999 + 0.3968 + 9.52$$

$$= 19.54\%$$

The density of a cylindrical rod was determined by $d = \frac{m}{\pi r^2 l}$. The % errors in m, r and l are 2%, 1.5% and 0.8% resply. Cal. the max per. error in the determination of d.

$$\frac{\Delta d}{d}\times 100\% = \frac{\Delta m}{m}\times 100 + \frac{\Delta l}{l}\times 100\% + 2\frac{\Delta r}{r}\times 100\%$$

$$= 2\% + 0.8 + 2\times 1.5$$

$$= 2 + 3 + 0.8 = 5.8\%$$

H.W.

13. A Physical Q. is related to observations using

$$P = \frac{a^3 b^2}{\sqrt{c}\, d}$$

The % errors of measurements in a, b, c and d are 1 %, 3 %, 4 % and 2 %. Find % error in P.

$$\frac{\Delta P}{P}\% = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}.$$

$$= 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\%$$

$$= (3 + 6 + 2 + 2)\% = 13\%$$

Q. The temp. of two bodies measured by two thermometer are $t_1 = 25° \pm 0.5°c$ and $T_2 = 20° \pm 0.2°c$. Cal. the temp. diff. and the error there in.

$$T_2 - T_1 = (25 - 20)°c = 5°c$$

$$\Delta T = \Delta T_1 + \Delta T_2. = 0.5 + 0.2 = 0.7.$$

$$T_2 - T_1 = (5°c \pm 0.7°c)$$

Significant figures.

The significant figures are the no. of digits upto which we are sure about their acuracy.

Rules.

1. All digits in a no. are significant.

   304 → 3

   3040 → 3.

2. In a ~~dig~~ no. without decimal point trailing zeros are not significant.

   200 - 1

   2010 → 3.

3. In a no. with decimal point, the trailing zeros are significant. 2.00 → 3 3.010 → 4

4. The zeroes to the right of the decimal point but to the left of first non zero digit are not significant.

   0.01030 → 4

   0.0001 → 1

   1.0010 → 5.

Significant figures in ~~arithmetic~~ algebraic operations.

1. Addition / subtraction.

When we add two like quantities then the final result cannot be more acurate than the least accurate measurement.

ie, the sum of the quantities should have the same no. of digits to the right of the decimal as the measurement with the smallest no. of. digits to the right of the decimal.

Ex.
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  12.32+
 100.1
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 112.42  →  112.4
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2. Multiplication / Division.

The no. of significant figures in the product / Quotient is equal to the least no. of significant figures in any of the numbers.

$x = 4.02$. $y = 2$.

$xy = 8.04$

$= 8$.