MECHANICAL PROPERTIES OF FLUIDS

Pressure - It is the force acting per unit area.
$P = F/A$. unit is N/m^2 or Pascal (Pa).

Applications - Railway tracks are laid on large sized wooden / concrete sleepers. - In the absence of wooden sleepers the weight of the train acts on the ground through rails of less area and pressure on the ground will be more.

2. One can walk on sand easily by placing wooden planks on the sand. Why?

Pressure exerted by a liquid of density ρ.

To find the pressure exerted by the liquid at a point A at a depth h from the surface, consider a column of liquid in the shape of a cylinder of area of cross section A and height h.

$$\text{Pressure at A} = \frac{F}{A} = \frac{mg}{A} = \frac{V \times \rho g}{A}$$

$$= \frac{A \times h \times \rho g}{A} = h\rho g$$

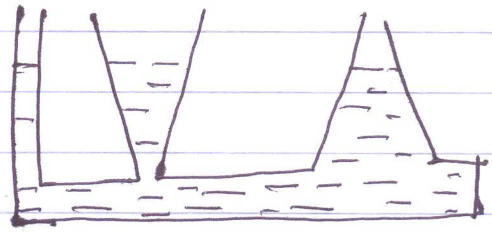
ie, pressure exerted by a liquid depends on the depth, its density and acceleration due to gravity 'g'.

Q. The base of the dams are constructed as thick. Why?

$P = h\rho g$. As depth of water increases, pressure also increases. To withstand the liquid pressure, the base of dams should be thick.

Pascal's law The pressure in a fluid at rest is the same at all ~~of~~ points if they are at the same height.

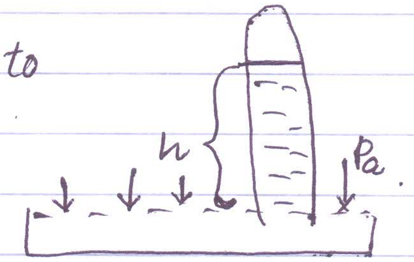
Hydrostatic paradox The water at the bottom of vessels A, B and C exerts the same pressure even if the area of the base is different. That is pressure due to a liquid is independent of area of cross section.



Atmospheric pressure and gauge pressure.

Atmospheric pressure is due to the collision of atmospheric molecules on a surface. It can be measured using a mercury barometer as shown.

It is seen that pressure due to the atmosphere can balance pressure due to 76 cm of mercury.



1 atmosphere = 76 cm of mercury

$= h\rho g = 0.76 \times 13600 \times 9.8$

1 atmosphere $= 1.01 \times 10^5$ Pa

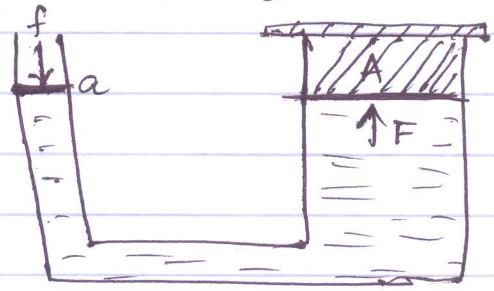
Pascals law for transmission of fluid pressure

Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions. This is the pascals law for transmission of fluid pressure. It has many applications in daily life. Hydraulic lift and hydraulic brakes are based on it.

In a hydraulic lift two pistons are separated by the space filled with a liquid. A piston of small crosssection 'a' is used to exert a force 'f' directly on a liquid. The pressure, $P = f/a$ is transmitted throughout the liquid and exert a force on the other piston of large area of cross section A. Let the force be F.

$$F = P \times A = \frac{f}{a} \times A = f \frac{A}{a}$$

ie, applied force can be increased by a factor $\frac{A}{a}$. This factor is the mechanical advantage of the device.



gauge pressure

Total pressure at a depth 'h' of a fluid of density ρ is,

$$P = P_a + h\rho g$$

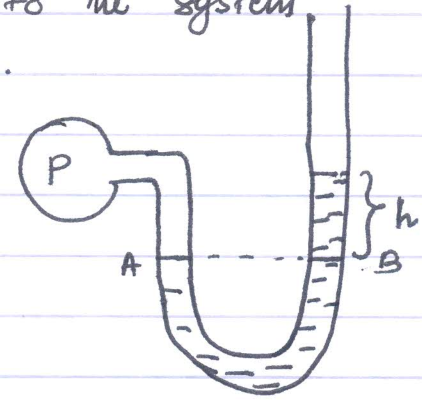
The excess pressure $P - P_a = h\rho g$ is called gauge pressure at that point.

Open-tube manometer - is an instrument for measuring pressure differences. It consists of a U tube containing a suitable liquid. One end of the tube is open to the atmosphere and the other end is connected to a system whose pressure is to be measured.

Pressure at A = pressure due to the system

= Pressure at B.

$= P_a + h\rho g$.



$$P - P_a = h\rho g$$

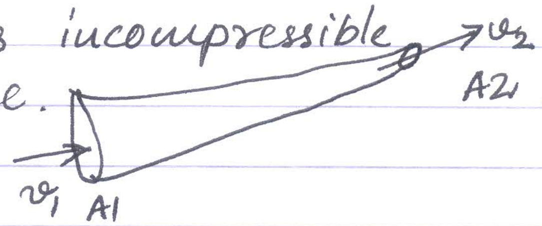
$$P - P_a \propto h$$

Stream line flow refer text page 253.

Reynolds number The velocity of a liquid flowing through a tube is given by,

$V = \frac{R_e \eta}{a\rho}$ where η - coeft. of viscosity, ρ the density of the fluid, and Rc is a dimensionless constant which decides whether the flow is streamlined / laminar or turbulent. It is called Reynolds number. If $Rc < 1000$, the flow is streamlined and if $Rc > 2000$, the flow will become turbulent. a - is the diameter.

<u>Equation of continuity</u> - Consider a portion of a narrow tube of two crossections A_1 & A_2. Since the liquid is incompressible, the density doesnot change.



The mass of the liquid entering the tube = mass of the liquid leaving the tube.

volume of liquid entering × ρ = Volume of the liquid leaving × ρ

$$Area_1 \times dx_1 = area_2\, dx_2.$$

$$A_1 v_1 dt = A_2 v_2 dt. \qquad dx_1 = v_1 dt.$$

$$A_1 v_1 = A_2 v_2 \qquad dx_2 = v_2 dt.$$

This equ is known as equation of continuity. which says that area of crossection of the pipe and the speed of flow of the liquid are inversely proportional to each other $A \propto 1/v$.

<u>Bernoulli's principle</u>

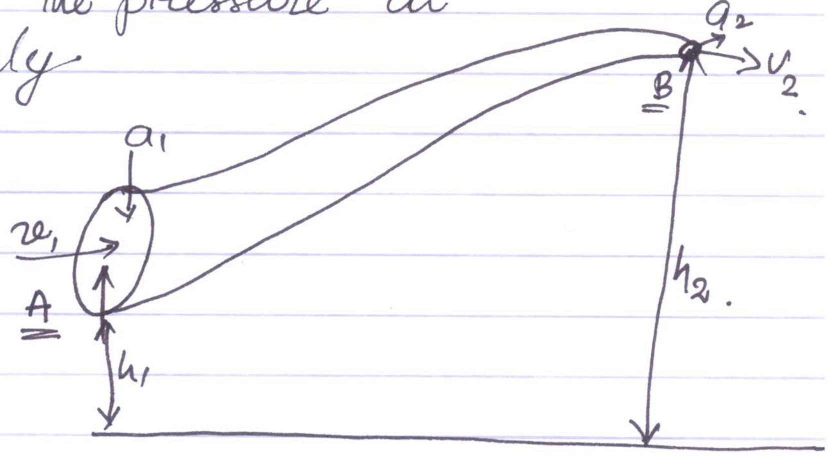
It states that the total energy, which is the sum of potential energy, kinetic energy and pressure energy of an incompressible liquid flowing from one point to another remains constant through out.

The assumptions made are,

1. the fluid is incompressible (density doesnot change)

2. the fluid is non-viscous.
3. the flow is steady.

Consider an incompressible liquid of density ρ flowing through a pipe of varying area of crosssection. Liquid enters at the wide end of cross section a_1 with vel. v_1 and leaves the narrow end of cross section a_2 with vel. v_2. Let the wide end is at a height h_1 and narrow end at a height h_2. Let P_1 and P_2 be the pressure at A and B respectively.



A liquid in motion have energy in three forms.

1. Potential energy

$$P.E = mgh.$$

$$P.E/\text{unit vol} = \frac{mgh}{V} = \rho g h$$

2. Kinetic energy $= \frac{1}{2}mv^2$

$$\text{Kinetic energy/unit vol} = \frac{1}{2}\frac{mv^2}{V} = \frac{1}{2}\rho v^2$$

3. <u>Pressure energy</u>

It is the energy possessed by a liquid due to its pressure. It is defined as the work done to move a liquid element from a point to another point at a depth x.

$$\text{Pressure energy} = \text{work done} = \int F \cdot dx = \int P \cdot A\,dx$$

$$= P\int dV = PV.$$

$$\text{Pressure energy/unit volume} = \frac{PV}{V} = P$$

In the diagram, as the liquid moves from A to B, its K.E and P.E increases and pressure energy decreases.

According to the law of conservation of energy
decrease in pressure energy = gain in K.E + gain in P.E.

decrease in pressure energy = $P_1 - P_2$.

gain in K.E = $K.E_f - K.E_i = \frac{1}{2}\rho(v_2^2 - v_1^2)$

gain in P.E = $P.E_f - P.E_i = \rho g(h_2 - h_1)$

Substituting these,

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

ie, sum of pressure energy/unit vol, the K.E/unit volume and P.E/unit vol will remains as a constant

If there is no height difference,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Applications 1. Lift of aeroplanes - As the speed of air above the aeroplane increases, the pressure decreases. Hence the pressure below the plane will exert a force ($F = \Delta P \times A$) on the plane and hence it lifts.

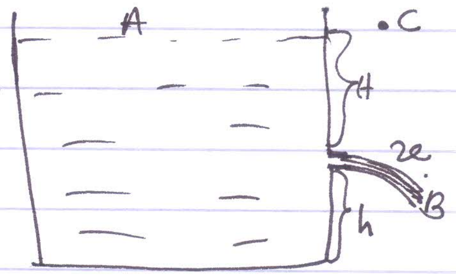
2. During storm, roof of small huts are blown off.

Speed of air above the huts becomes more during thunderstorms and pressure reduces. Due to high pressure below the roof, an upward force acts and it takes away the roofs.

Torricelles theorem - The speed of efflux (the speed of flow of a liquid) from an open tank is the same as the velocity gained by a freely falling body through the same height.

Proof

Consider a tank containing a liquid of density ρ. with a small hole in its side at a height h from the bottom.



Applying Bernoulli's theorem at A an B,

$$P_a + \frac{1}{2}\rho \times 0 + \rho g(H+h) = P_a + \frac{1}{2}\rho v^2 + \rho gh.$$

where v is the speed of efflux.

$$\rho g(H+h) = \frac{1}{2}\rho v^2 + \rho gh.$$

$$\rho g H = \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gH} \quad \text{—— (1)}$$

Consider a freely falling body from C. $u = 0$,

$$v^2 = u^2 + 2as.$$

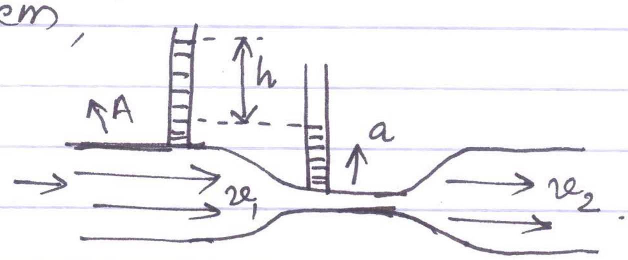
$$v = \sqrt{2gH} \quad \text{—— (2)}.$$

ie, speed of efflux = speed gained by a freely falling body.

Ventury-meter – It is a device to measure the flow speed of incompressible fluid. It consists of a tube with a broad diameter with a small constriction at the middle. A manometer is also attached to it with a liquid of density ρ_m. Let A be the area of crosssection of the wide neck and a the area of crosssection of narrow neck. Let v_1 be the speed at the wide end and v_2 be the speed at the narrow end.

Using the equ. of continuity $Av_1 = av_2$; $v_2 = \frac{A}{a}v_1$

Using bernoulli's theorem,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho \frac{A^2}{a^2} v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A^2}{a^2} - 1\right)$$

This pressure difference causes the fluid in the

U tube connected at one end to rise in comparison to the other arm. The difference in height 'h' measures the pressure difference ie, $P_1 - P_2 = h\rho_m g$

$$h\rho_m g = \frac{1}{2}\rho v_1^2\left(\frac{A^2}{a^2} - 1\right)$$

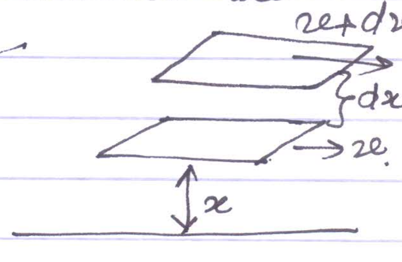
$$v_1^2 = \frac{2h\rho_m g}{\rho\left(\frac{A^2}{a^2}-1\right)} \qquad v_1 = \left[\frac{2gh\rho_m}{\rho\left(\frac{A^2}{a^2}-1\right)}\right]^{1/2}$$

For more applications of Bernoulli's principle, refer text page 257 - 258.

VISCOSITY.

The property of a liquid by which a resistance comes into play, when a liquid is in motion is called viscosity.

Cause of viscosity. The layer of the fluid which is in contact with the ground will be at rest. As the distance from the ground increases, the velocity of different layers also increases. Thus there is a relative motion between different layers. This causes the frictional force come into play.



The velocity gradient $= \frac{dv}{dx}$.

It is the change in velocity with distance between two layers of fluid flow

~~velocity~~ = 0

Laminar flow - A flow that can be considered as made up of different layers.

Newton's law of viscosity.

The viscous force F between two different layers is directly proportional to the area A of the liquid layer and the velocity gradient.

$$F \propto A\frac{dv}{dx}$$

$F = -\eta A \frac{dv}{dx}$ where η (eta) is called the coefficient of viscosity of the liquid. It depends on the nature of the fluid.

$$\eta = F/A \frac{dv}{dx}$$

ie, coeft. of viscosity is defined as the tangential force required / unit area to maintain unit velocity gradient between two parallel layers.

S.I. unit Poiseiulle (Nsm^{-2} or Pa s)

H.W. Find the dimensions of 'η'.

Coefficient of viscosity 'η' is inversely proportional to temperature.

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