UNIFORM CIRCULAR MOTION.

When an object moves in a circular path at a constant speed the motion of the object is called uniform circular motion.

Uniform circular motion is accelerated. Even if the speed is constant, the direction of velocity continuously changes. Hence it is accelerated. This acceleration is called centripetal acceleration. The direction of centripetal acceleration is towards the centre of the circle along the radius.

Angular displacement ($\Delta\theta$.) - is the angle described by the position vector in Δt time.

From the figure,

$$\Delta\theta = \frac{AB}{r}.$$

S.I. unit - radian.

$$\pi \text{ rad} = 180^\circ.$$

Angular velocity (ω) - rate of change of angular displacement is called angular velocity ω.

$$\omega = \frac{\text{angular displacement}}{\text{time}} = \frac{\Delta\theta}{\Delta t}.$$

It is a vector. S.I. unit rad s^{-1}. Dimension $M^0L^0T^{-1}$

Relation between linear velocity (v) and angular velocity ω.

Consider an object moving in a circular path of radius 'r' with a constant speed 'v'. Let it describe an angle $\Delta\theta$ in time 'Δt'.

From the fig $\Delta\theta = \frac{AB}{r}$

$\div$ by Δt, $\frac{\Delta\theta}{\Delta t} = \frac{\Delta x}{r\,\Delta t}$

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{r} \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$

$$\omega = \frac{1}{r}.v$$

$$\underline{\underline{v = r\omega}}$$

Angular Acceleration (α)

It is the rate of change of angular velocity $\alpha = \frac{d\omega}{dt}$.

Unit - $rad s^{-2}$ Dimension - $M^0 L^0 T^{-2}$.

Relation between angular accelerations and linear acceleration a

We have $a = \frac{dv}{dt}$ But $v = r\omega$.

$$= \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt}.$$

$$\boxed{a = r\alpha}$$

Time period (T) - Time taken by an object to complete one revolution. unit - second.

Frequency (ν) - number of revolutions per second.

Unit - Hertz (Hz).

$$T = \frac{1}{\nu}$$

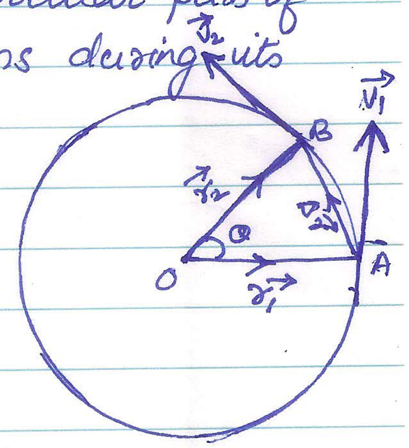
Relation between angular velocity (ω) & frequency

$\omega = \frac{\Delta\theta}{\Delta t}$ For one complete revolution, $\Delta\theta = 2\pi$, $\Delta t = T$.

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Derive an expression for centripetal acceleration (a_c)

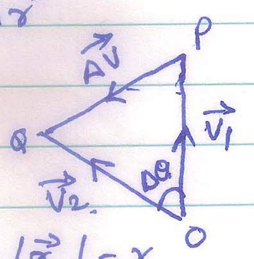
Consider an object moving in a circular path of radius 'r'. Let 'A' and 'B' be two positions during its motion.



Using triangle law $\vec{r_2} = \vec{r_1} + \Delta\vec{r}$

$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

Drawing velocity vectors,



$\vec{v_2} = \vec{v_1} + \vec{\Delta v}$

$\vec{\Delta v} = \vec{v_2} - \vec{v_1}$ Also, $|\vec{r_1}| = |\vec{r_2}| = r$

In Δ OPQ, and $|\vec{v_1}| = |\vec{v_2}| = v$

$\Delta\theta = \frac{\Delta v}{v}$ $\div$ by Δt, $\omega = \frac{v}{r}$

$\frac{\Delta\theta}{\Delta t} = \frac{1}{v}\frac{\Delta v}{\Delta t}$, $\omega = \frac{a_c}{v}$, $a_c = v\omega = \frac{v.v}{r} = \frac{v^2}{r} = r\omega^2$

Also $F = ma_c = \frac{mv^2}{r} = mr\omega^2$.